Quadratics and exponents and logs...oh my!

(Actually, the first part is just a reminder about completing the square. But I said "quadratics" in the title because then I could make the Wizard of Oz joke. It wouldn't work the same with "completing the square." But moving on!)

Completing the square: the idea with completing the square is that every single quadratic $Ax^2 + Bx + C$ is a perfect square plus some constant. That is:

$$Ax^2 + Bx + C = a(x+b)^2 + c$$

Completing the square is all about finding those a, b, and c for the particular quadratic you are given. How do we find them? Here's the process that never fails:

- Expand $a(x+b)^2 + c$
- Match coefficients on both sides of the equation
- Solve for a, b, and c

Algebraically, this means:

$$\begin{aligned} Ax^2 + Bx + C &= a(x^2 + 2bx + b^2) + c \\ Ax^2 + Bx + C &= ax^2 + 2abx + ab^2 + c \end{aligned} \qquad \Longleftrightarrow \qquad \begin{aligned} A &= a \\ B &= 2ab \\ C &= ab^2 + c \end{aligned}$$

Completing the square can be useful in sketching quadratic functions, determining when $Ax^2 + Bx + C = 0$, and certain integration problems we will see in Math 20.

Exponent laws: Important facts to remember about exponents in the case where a and b are real numbers.

$$x^a x^b = x^{a+b} \qquad (x^a)^b = x^{ab} = \left(x^b\right)^a \qquad \frac{x^a}{x^b} = x^{a-b}$$

$$(xy)^a = x^a y^a \qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \qquad x^{-a} = \frac{1}{x^a}$$

It's also important to remember that $x^0 = 1$ for any finite number x and $x^1 = x$.

In the case where the exponent is $\frac{1}{n}$ for some positive integer n, we get the idea of taking a root. That is

$$x^{1/n} = \sqrt[n]{x}$$

From our general exponent laws above, we get the following laws about these roots:

$$\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y} \qquad x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m \qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Logarithms: the logarithm base a is the inverse of exponentiation. That is:

$$\log_a x = y \iff x = a^y$$

The domain of a logarithm function is $(0, \infty)$ and the range is $\mathbb{R} = (-\infty, \infty)$. Working with the fact that logarithm inverts exponentiation and the exponent laws above, we have the following laws about logarithms:

$$\log_a(xy) = \log_a x + \log_a y \qquad \log_a(x^b) = b \log_a x \qquad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

The most common base for log that we will see in this course is e, which gives us the natural logarithm. It's so special, it gets its own notation: we write $\ln(x)$ to mean $\log_e x$.

CAUTION: common math mistakes

$(x+y)^a \neq x^a + y^a$	all of these are NOT EQUALS signs	$\frac{1}{a+b} \neq \frac{1}{a} + \frac{1}{b}$
$\log_a(x+y) \neq \log_a x + \log_a y$		$\sqrt[n]{x+y} \neq \sqrt[n]{x} + \sqrt[n]{y}$